Modulated Double-Helix Quadrupole Magnets

R. B. Meinke, C. L. Goodzeit, and M. J. Ball

***Abstract*— We describe a new technology for superconducting quadrupole magnets especially for use in particle accelerators. The principle is based on the application of a sinusoidal modulation to the axial positions of the conductor windings in solenoids. The method can also be employed to produce higher-order multipole fields. Due to their solenoid-like geometry, these coils are significantly simpler to manufacture than standard (racetrack) cosine-2-theta coils and have significantly smaller systematic field errors without using any field-shaping spacers. When two complementary coil layers (with opposite winding directions and current flow) are combined, the solenoid components of the fields are cancelled and the quadrupole or higher-order fields add. An example of such a design is described which generates a gradient of 130 T/m with systematic errors less than 10^-8 at 67% of the aperture.**

***Index Terms*—Accelerator magnets, Superconducting quadrupoles**

I. Introduction

The method of modulation of a helical conductor path to create a quadrupole field in the aperture can be explained by first referring to the geometry of a tilted elliptical loop as shown in Fig. 1.

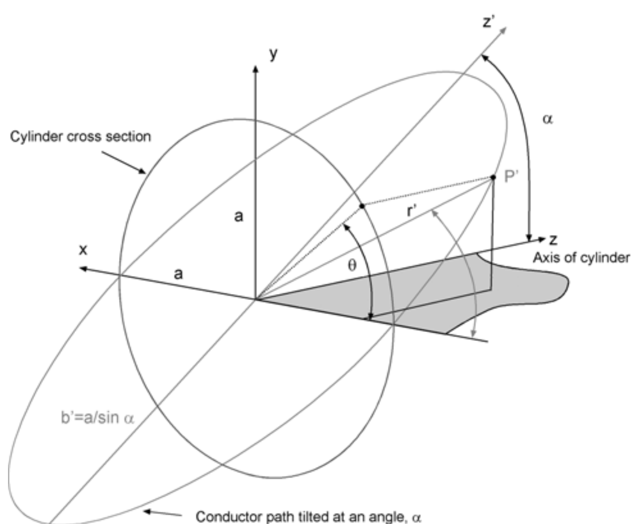

Fig. 1. Coordinates of a tilted elliptical loop.

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The loop is tilted at an angle α with respect to the z-axis. A point P at an angle θ on the circular aperture has a projection P' on the conductor path in the tilted plane. The distance between P and P' in the z-direction is given as $z(\theta)$.

The z-coordinate of the point P' on the loop at an angle θ is:

$$z(\theta) = \frac{a}{\tan\alpha}\sin\theta = A_0 \sin\theta \qquad (1)$$

A_0 can be considered to be amplitude of a sinusoidal modulation function of order n = 1 and is determined from the tilt angle of the loop since $A_0 = \frac{a}{\sin\alpha}$. Considering helical loops with an advance h in the helical winding per turn, as shown in Fig. 2, and generalizing (1) to use a modulation function of order n, the z-coordinate of the current path can be expressed as

$$z(\theta) = \frac{h\theta}{2\pi} + A_0 \sin n\theta \qquad (2)$$

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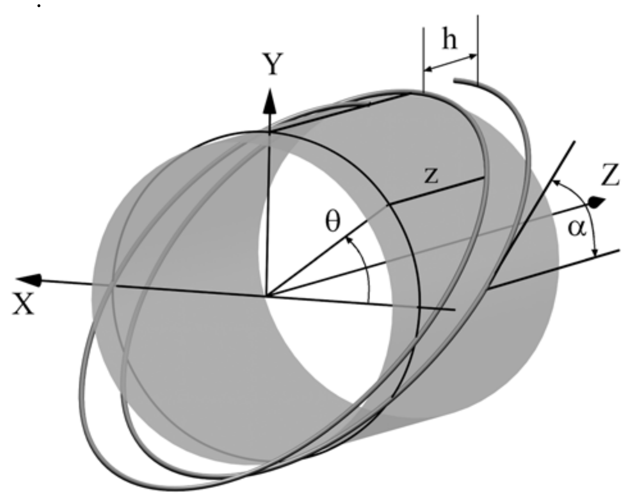

Fig. 2. Coordinates of tilted helical loops with modulation n=1 (dipole)

The case for n=1 represents a sinusoidal modulation of the z-coordinate at a frequency of once per turn. This case represents the descriptive geometry of a *double-helix dipole* [1], which produces a pure dipole field (multipole order 0) when 2 layers of opposite tilt angle and current flow are used to cancel the solenoid field.

It follows that when n=2,3, etc., pure multipole fields of order n-1 can be produced by using complementary pairs of concentric helical coils with opposite winding direction and current flow.

For n=2, the configuration for one coil layer produces a helical space curve modulated as shown in Fig. 3. When two complementary coils (one wound CW and one wound CCW) are assembled concentrically with opposite direction of current flow, as shown in Fig. 4, the result is a quadrupole magnet (the double-helix quadrupole or DHQ).

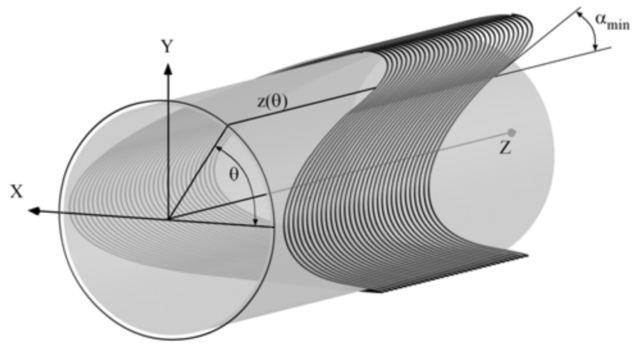

Fig. 3. Helical turns modulated with sine 2θ.

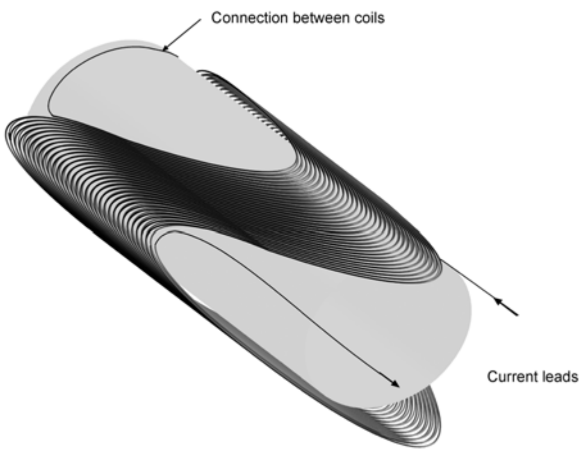

Fig. 4. Coil arrangement for a double-helix quadrupole (DHQ). Partial length coils are shown, to illustrate both lead and return ends of winding.

II. MAGNETIC FIELD PROPERTIES

A. Field Strength and Gradient

For the case without iron or end effects, the magnetic field strength produced by each layer of a modulated helical coil (as shown in Fig. 3) can be estimated from the geometrical parameters of a series of closed modulated helical current loops, much in the same way as has been done for the dipole configuration [1]. The minimum number of effective turns/meter in the azimuthal direction provides the magnetizing force for the maximum tangential field component and the number of effective turns/meter in the axial direction determines the solenoid (z-direction) magnetizing force.

The turn spacing is related to the effective conductor width d and the angle that the turn makes with respect to the z-axis. Using (2) for the three-dimensional space curve of the closed loop in Fig. 3, the z coordinate of a point on the loop can be written in terms of the circumferential distance from the mid-plane as

$$z(p) = A_0\left(1-\cos\left(\frac{2p}{a}\right)\right) \qquad (3)$$

where $p = a\theta$ is the distance along the circumference of the coil aperture. The minimum turn spacing that can be achieved with a series of loops of effective conductor width d can then be determined from the minimum angle that the space curve makes with the z-axis.

The slope of the space curve with respect to the z-axis is given by $\frac{dz}{dp} = \frac{2A_0}{a}\sin\left(\frac{2p}{a}\right)$. For the coil orientation shown, this has a maximum of $\frac{2A_0}{a}$ when $p = \frac{\pi}{4}a$ and, thus, the minimum angle between the loop turn and z-axis is:

$$\alpha_{min} = \tan^{-1}\left(\frac{\pi}{2} - \frac{2A_0}{a}\right) = \tan^{-1}\left(\frac{a}{2A_0}\right) \qquad (4)$$

The minimum spacings for the turns are thus:

$$h = \frac{d}{\sin\alpha_{min}} \text{ (axial) and } v = \frac{d}{\cos\alpha_{min}} \text{ (circumferential)}$$

which gives the turn density in the circumferential direction as $\frac{1}{v} = \frac{\cos\alpha_{min}}{d}$ (turns per meter of aperture). Hence, the tangential and radial quadrupole transfer functions (field components per ampere) at r = a and $p = \frac{\pi}{4}a$ are:

$$\chi_{\theta\max} = \frac{\mu_0}{2d}\cos\alpha_{min} \quad \text{and} \quad \chi_r = 0 \qquad (5)$$

Within a quadrupole coil, the amplitude of the field scales linearly with the radial distance but does not depend on the angular position. Thus, the amplitude of the quadrupole transfer function (denoted by χ) can be determined from the maximum for the tangential field component, which occurs at r=a where the radial field component is zero. Thus,

$$\chi = \chi_{\theta\max} = \frac{\mu_0}{2d}\cos\alpha_{min} \qquad (6)$$

The solenoid field per ampere for one coil layer is given by:

$$\chi_z = \frac{\mu_0}{d}\sin\alpha_{min} \qquad (7)$$

Using (4) with $\alpha_{min} = \alpha$, we get

$$\cos\alpha = \frac{2A_0}{\sqrt{4A_0^2 + a^2}} \text{ and } \sin\alpha = \frac{a}{\sqrt{4A_0^2 + a^2}} \qquad (8)$$

Using (6) for the transfer function at r=a and scaling the field with the radial dependence, the general expression for the quadrupole field per ampere at radius r is:

$$\chi_1 = \frac{A_0\mu_0}{d\sqrt{4A_0^2 + a^2}}\left(\frac{r}{a}\right) \qquad (9)$$

If we denote the ratio of the modulation amplitude to the coil radius by $k = \frac{A_0}{a}$, then the strength of the field per ampere at r = a is:

$$\chi_1 = \frac{k\mu_0}{d\sqrt{4k^2+1}} \quad \text{where } k = \frac{A_0}{a} \text{ and } g = \frac{\chi_1}{a} \tag{10}$$

From (7), the solenoid field component for a single coil is:

$$\chi_z = \frac{\mu_0}{d\sqrt{4k^2+1}} \tag{11}$$

The above relations are for a single layer coil. The cancellation of the solenoid field component by pairs of complementary concentric coils (i.e., coils with opposite winding direction and current flow) is accomplished if the axial turn spacing is the same in each layer and the exact same number of turns is used in each layer. Thus, the value of k must remain a constant for each concentric pair of coils in the magnet. Consequently, for a multiple layer coil of N layers (N=2, 4, 6,..) with radii a_N, the field per ampere at the radius r=a of the innermost coil is

$$\chi_1 = \frac{k\mu_0}{d\sqrt{4k^2+1}}\left(1+\sum_{n=2}^{N}\frac{a}{a_n}\right) \tag{12}$$

and the gradient is $\quad g = \frac{\chi_1}{a}$ (13)

B. *Field Enhancement in the Double-Helix Quadrupole*

Since the peak field seen by the conductor limits the performance of the magnet, it is of interest to determine the peak field in the DHQ configuration. The solenoid field from the concentric pairs of coils is cancelled everywhere except between the coil layers that make up a pair of complementary coils. Thus, the peak field per ampere can be estimated by calculating the quadratic sum of the transfer functions for the maximum tangential field seen by the inner most layer (6) and one layer's solenoid field component (7). This yields the peak field per ampere:

$$\chi_p = \frac{\mu_0}{d\sqrt{4k^2+1}}\sqrt{k^2\left(1+\sum_{n=2}^{N}\frac{a}{a_n}\right)^2+1} \tag{14}$$

from which the field enhancement is

$$E = \frac{\chi_p}{\chi_1} - 1 \tag{15}$$

The field enhancement can be controlled by using modulation factors of 1.0 or greater and multiple layer coils. The example that we show in this paper uses a four-layer coil with a modulation factor of 1.0 to achieve a field enhancement of 4.85%.

C. *Obtaining Error-free Fields*

Ignoring end effects and excluding the presence of iron, we should be able to use the modulation concept of (2) with n=2 and higher modulation factors to design helical coils that will have error-free fields for any desired multipole.

This has been verified by using numerical calculations with CoilCAD for several cases of concentric pairs of complementary modulated coils. Fig. 5 is the CoilCAD output for the double-helix quadrupole case, n=2.

An and Bn are the skew and normal multipole coefficients of the total field. Cn = sqrt (An^2 + Bn^2), is the strength of the n-pole field. The three-dimensional components of the field are BX (horizontal), BZ (vertical, dipole), and BY (axial, solenoid).

The non-vanishing A0, B0 (dipole) terms have been shown to be related to the number of integration points used in the model. These terms tend to go to zero as the number of integration points is increased.

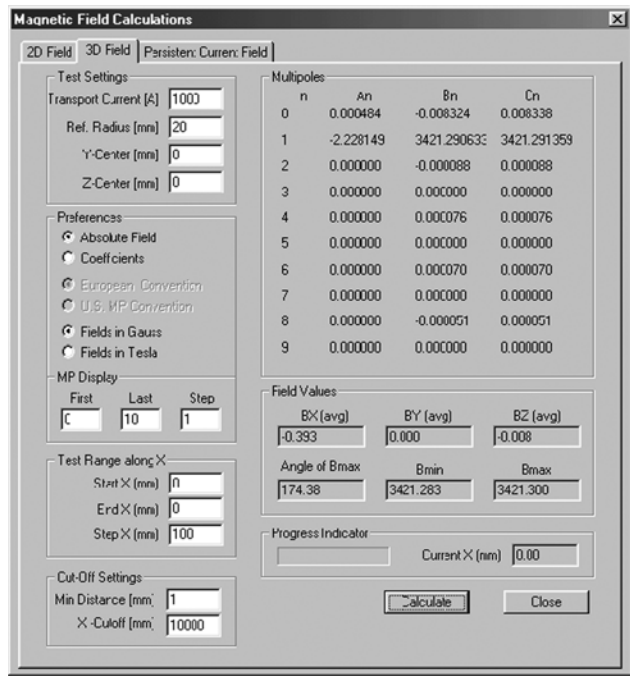

Figure 5. CoilCAD results for a DHQ coil (n=2)

III. Example DHQ Design (60 mm Aperture, 130 T/m)

A. *Coil Parameters*

As a design example, a 60 mm aperture double-helix quadrupole coil is used. We consider the case of a coil without end effects and without the presence of iron. A configuration is described which consists of a 4-layer coil with an aperture of 60 mm and modulation amplitude of 30 mm for the inner-most coil as shown in Fig. 6. A 19-strand NbTi superconducting cable is chosen with the properties shown in Table I.

Table I. Properties of Superconducting Cable

Strand diameter, mm	0.808
Number of NbTi strands	19
Cu:NbTi	1.3
Cable diameter, bare ,mm	4.04
Cable diameter, insulated, mm	4.54

Cable I_c at 4.2 K and 6 T, A	9654

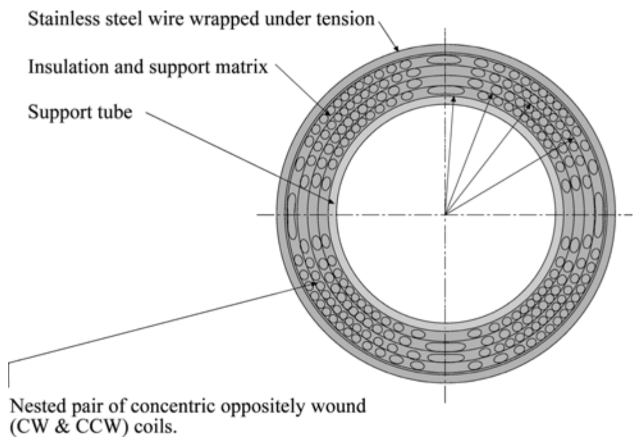

Fig. 6. 130 T/m gradient quadrupole example.

Table II summarizes the design and operating parameters for this magnet that produces a gradient of 130 T/m in the 60 mm aperture.

TABLE II. PARAMETERS FOR EXAMPLE MAGNET.

Coil radii (r_1, r_2, r_3, r_4), mm	30.00, 35.54, 41.08, 46.62
Effective conductor width (d), mm	4.54
A0 for r_1, mm	30
Modulation factor (k)	1
Operating temperature, K	4.35
Operating current, A	9790
Quench current, A	11,819
Operating current margin	17.1%
Field at r_1 in x-y plane, T	3.90
Quadrupole gradient, T/m	130
Solenoid field component between coil pair layers, T	1.21
Peak field on conductor, T	4.08

B. Field errors

For the example shown, it is expected that the design should produce a pure quadrupole field in the coil aperture. This has been determined to be the case based on a CoilCAD numerical analysis of the above configuration, which showed that the systematic field errors are all less than 10^{-8} of the main field component at 67% of the coil aperture.

IV. MAGNET MANUFACTURING CONSIDERATIONS

A. Simpler Construction

The manufacture of the modulated double-helix quadrupole configuration is expected to be considerably less expensive than the cost for comparable-performance quadrupoles made of conventional racetrack-configuration coils.

The tilted-helix geometry allows the coils to be fabricated using standard machine tools in much the same way that high-field solenoid coils are made. Thus, model and prototype double-helix coils can be made with minimum tooling investment.

Another major cost-saving advantage occurs from the reduced number of coils and parts that are required to complete the DHQ assembly.

B. Combined Function Magnets

The modulation technique described herein can be extended to include the generation of superimposed multipoles to produce *combined function magnets without unwanted multipole components*. For example, if the z(θ) coordinate of the conductor path is modulated by a function of the form

$$z(\theta) = \frac{h\theta}{2\pi} + A_0 \sin\theta + A_1 \sin 2\theta$$

the result will be a combined function dipole and superimposed quadrupole. Other multipole combinations comprising higher order terms are equally possible by using modulation terms of the form $A_{n-1} \sin(n\theta)$.

V. SUMMARY

The concept of modulating the axial position of a helical conductor path in a solenoid with a sinusoidal function of order n has been shown to produce a quadrupole field for n = 2. The case for n = 1 is a double-helix dipole which is described separately [1]. Higher multipole fields are possible for n = 3, 4, etc. Concentric pairs of complementary coils (with opposite winding direction and current flow) enhance the multipole field and cancel the solenoid field components of the two layers. Thus, magnets of any multipole order, without systematic error fields, are possible by using multiple nested pairs of such coils.

We have shown an example case for a superconducting quadrupole that produces a gradient of 130 T/m in an inner coil aperture of 60 mm. This magnet uses two pairs of complementary double-helix quadrupole coils.

The double-helix quadrupoles are manufactured in much the same way as the double-helix dipoles. Thus all of the advantages of simplified design and lower cost of manufacturing that were described in the paper [1] about the dipoles pertain to the quadrupoles as well.

Numerical analysis has shown that the modulation winding technique for helical coils (without iron and end effects) produces error-free multipole fields in a large fraction of the coil aperture. Thus, quadrupole magnets with very high field quality are possible using this technique. In addition, the technique lends itself to the production of combined function magnets.

REFERENCES

[1] C. L. Goodzeit, R. B. Meinke, and M. J. Ball, “The Double-Helix Dipole – A Novel Approach to Accelerator Magnet Design”, Paper 4LC07, ASC2002, Houston, TX, August 2002.