

## Combined Function Magnets Using Double-Helix Coils\*

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**Abstract:** We describe a technology for creating easy-to-manufacture combined function magnets. The field is produced by double-helix coils in which the axial path of the windings is defined by a sinusoidal function containing the superposition of the desired multipoles. The result is a magnet that can contain, for example, a pure dipole field with a superimposed quadrupole field whose magnitude relative to the dipole field can be easily controlled to any level. We show how low level (i.e. 0.1% – 1%) modulation amplitudes of the superimposed multipoles can be used as built-in or “free” correction coils to compensate for iron saturation effects or geometrically-induced multipoles. The combined function winding can also be used to superimpose higher amplitude multipoles such as a quadrupole whose integral of  $Gdl$  can be adjusted to any level desired over the length of the main dipole magnet. In this way a “free” quadrupole can be obtained with a dipole. The characteristics of this type of combined function magnet are also discussed.

### 1. FOREWORD

The superposition of higher order multipoles in iron-dominated dipole magnets has been previously obtained by shaping the iron pole pieces, such as for the Alternating Gradient Synchrotron at BNL and the Proton Synchrotron at CERN. In these cases, the iron was contoured to produce a quadrupole gradient superimposed on the dipole field. However, for the case of superconducting magnets whose field is mainly determined by the positions of the conductor, superimposed multipoles can be obtained using the double-helix design by a modification of the conductor path to produce the desired harmonic content. This procedure is a straightforward extension of the double-helix magnet technology and enables the easy manufacture of coils in which any order harmonic can be easily tuned to a specific level. This method is quite powerful since each harmonic can be tuned independently of other harmonics. When extended to the case that includes the non-linear effect of an iron yoke, this method can also be shown to be effective for compensating for saturation-induced multipoles.

### 2. DISCUSSION

#### *2.1. Double-Helix Coils and Superimposed Multipoles*

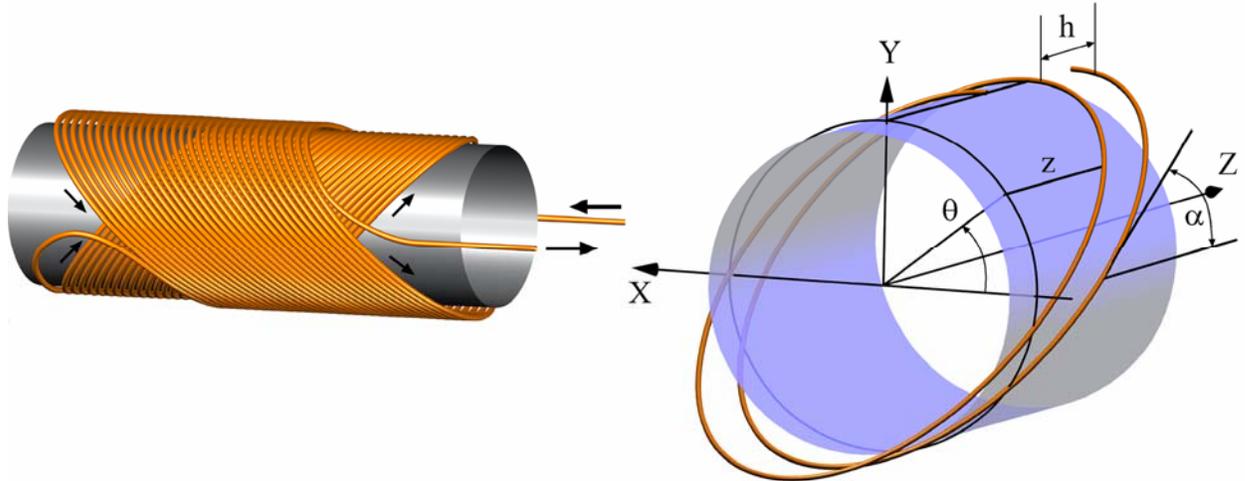
The double-helix dipole (DHD) and the double-helix quadrupole (DHQ) have been described in several papers [1,2,3]. The geometry of the conductor path that will produce a pure dipole field is shown in Figure 1. It is seen that each tilted helical turn has an advance  $h$  in the axial direction (the z-coordinate of the conductor path). It follows that, in the straight section of a long coil, pure multipole fields can be produced by pairs of oppositely-tilted coil windings with appropriate sinusoidal modulation of the axial position of the turns. Modulation according to  $\sin(n\theta)$ , with  $\theta$  being the azimuthal angle around the magnet axis, produces a multipole field of order  $n$ , where  $n=1$  is a dipole, as shown in Figure 1.

By using concentric pairs of this coil geometry with opposite tilt angle, the intrinsic solenoid field component is completely canceled, while the transverse multipole fields are

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additive. With a NbTi multi-strand superconductor operating at 4.35 K, this dipole configuration can produce about 2 T per 2-layer pair (without iron). Thus, multiple pairs of layers are required for higher fields.



**Figure 1. (Left) Layout of double helix winding. The axial field components of the 2 layers cancel each other and the total transverse field is enhanced. (Right) For the case of a dipole, the  $z$  coordinate of the conductor path is given by  $z(\Theta) = h \Theta / 2\pi + A_0 \sin \Theta$  with  $A_0 = a / \tan \alpha$  where  $a$  is the radius of the coil aperture,  $\alpha$  is the tilt angle of the winding with respect to the horizontal axis, and  $h$  is the helical advance per turn.**

The double helix winding concept can be readily extended to produce pure higher order multipole magnets, and as we shall show, combinations of superimposed multipole fields. This can be seen from the general expression for the conductor path of a double-helix coil given by:

$$z(\theta) = \frac{h\theta}{2\pi} + A_0 \left( \sin \theta + \sum_{n=2}^N \varepsilon_n \sin(n\theta + \phi_n) \right) \quad \text{Equation 1}$$

where the geometric variables are described in Figure 1.

The variable  $\varepsilon_n$  is the fraction of the dipole sinusoidal modulation,  $A_0$ , for the superimposed harmonic and the angle  $\phi_n$  of the  $n$ th harmonic is the phase angle between the harmonic and the fundamental dipole field. By controlling the phase angle, either a normal or a skew harmonic can be superimposed on the dipole field. The different multipole fields generated in this manner are completely orthogonal, i.e., including a field component of order  $n$  does not affect any other multipole order. This is a powerful method for correcting unwanted multipoles that arise in practical accelerator magnet applications such as bent dipoles. The bend radius and iron saturation effects can be tuned out by the inherent geometry of the coil winding and thus no additional correction coils or precise coil spacers (as used in the conventional magnet) are required.

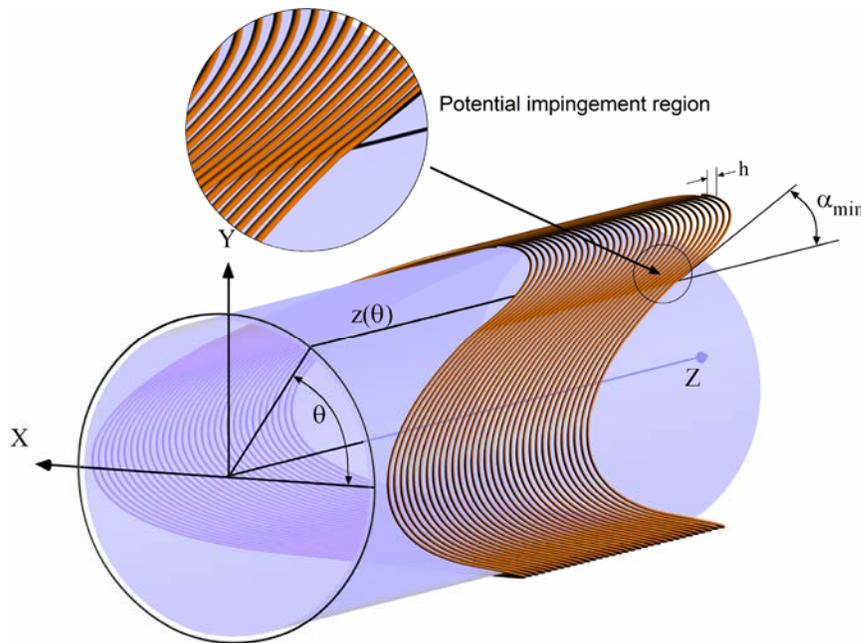
The same procedure can be used to form combined function magnets that contain a series of specified higher order harmonics. This effect can be illustrated by the addition of a superimposed quadrupole to a dipole field. Thus, Equation 1 for this case becomes:

$$z(\theta) = \frac{h\theta}{2\pi} + A_0 (\sin \theta + \varepsilon_2 \sin(2\theta + \phi_2)) \quad \text{Equation 2}$$

with  $\phi_2 = 0$  for a skew quadrupole,  $\phi_2 = 45^\circ$  for a vertical focusing normal quadrupole, and  $\phi_2 = -45^\circ$  for a horizontal focusing normal quadrupole.

### 2.2. Limitation of Multipole Content

However, the fraction,  $\varepsilon_n$ , of added multipole component is limited by the winding geometry. The modification of the conductor path for higher-order multipole terms means that the conductor tilt angle will vary around a turn. At the region where the tilt angle of the conductor with respect to the axial direction is a minimum, the conductor's z-direction thickness is a maximum and there is potential for conductor impingement. This is illustrated in Figure 2 for the case of a quadrupole winding.



**Figure 2. Potential impingement region of a double-helix winding for a quadrupole coil example. With an effective conductor width,  $d$ , the axial turn spacing has to be  $h \geq \frac{d}{\sin \alpha}$  in order to prevent the conductors from overlapping where the tilt angle,  $\alpha$ , is a minimum. This occurs at  $\theta = 45^\circ$  in a quadrupole and at the mid-plane ( $\theta = 0^\circ$ ) in a dipole.**

Thus, for large amounts of superimposed multipoles, especially the quadrupole, the turn spacing,  $h$ , needs to be increased to prevent any conductor impingement. This effectively lowers the transfer function of the dipole component of the coil. For small additions of the quadrupole (i.e.  $<0.1\%$  for correcting the effect in a bent magnet), the linear turn spacing can remain virtually unchanged. However, for a combined function magnet that contains a significant quadrupole content (i.e. 10 -50%), the turn spacing needs to be increased. This effect is shown in Figure 3 in which the dipole transfer function is plotted as a function of the quadrupole amplitude for both the normal and skew quadrupole. It is seen that the addition of the skew

quadrupole produces less of a decrease in the dipole transfer function. This is explained with reference to Figure 4.

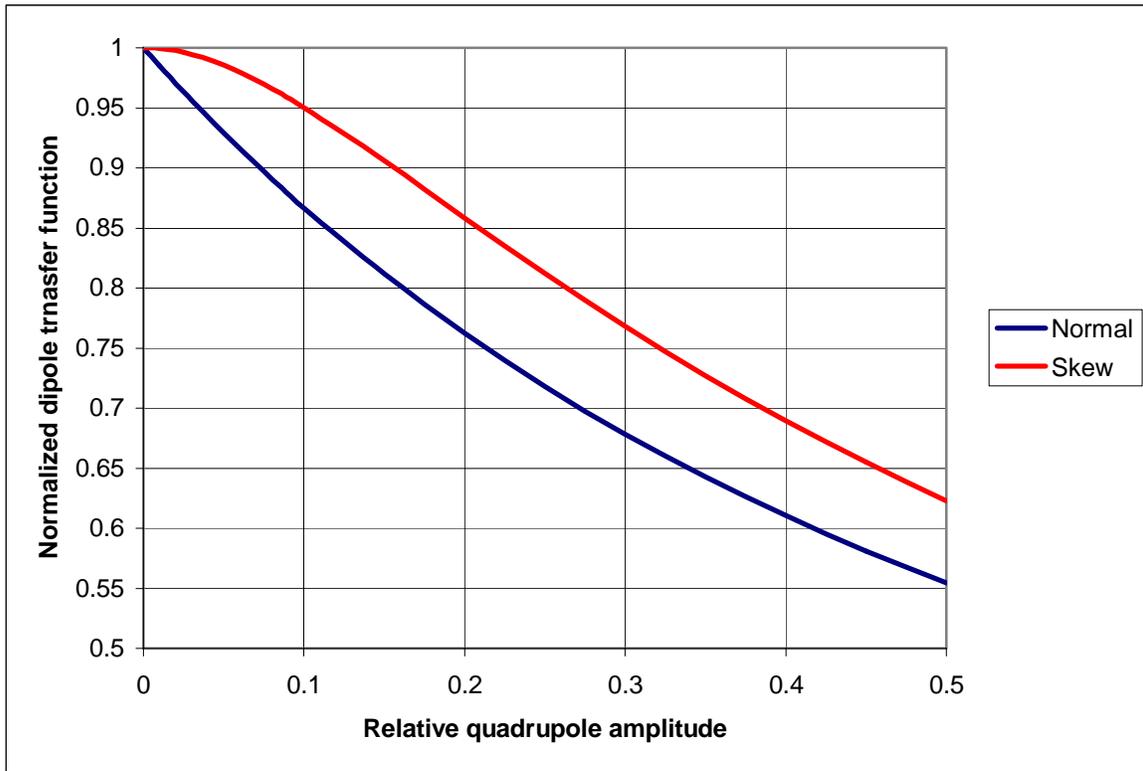


Figure 3. The dipole transfer function decreases with the amount of quadrupole amplitude because of the conductor impingement limitation. The difference between the normal and skew quadrupole come from the geometrical effect of the winding.

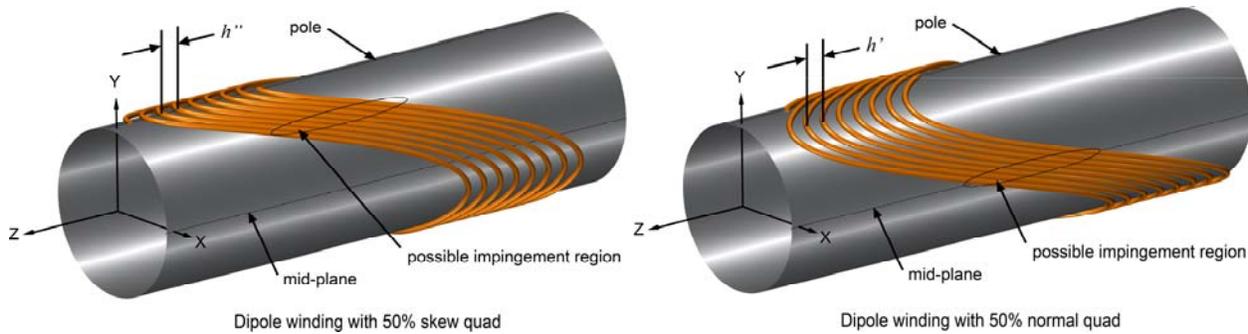


Figure 4. Dipole windings with 50% skew and normal quad amplitudes.

As shown in figure 4, the presence of a skew quadrupole component in the dipole winding causes the turns to come closest together at approximately 45° from the mid-plane since the basic skew quadrupole winding shown in Figure 2 produces that effect. Since the dipole turn spacing is not at its minimum (i.e. at the mid-plane), it is possible to decrease the axial turn spacing to a value of  $h'' < h'$  where  $h'$  is the minimum spacing for the case of the normal quadrupole superimposed on the dipole. When we apply this rule to the case of a dipole with a 50% horizontal focusing quad, we get the result shown in Table 1.

**Table 1. Parameters for a 4-layer double-helix dipole (with iron yoke) compared to the same magnet with 50% normal focusing quadrupole content.**

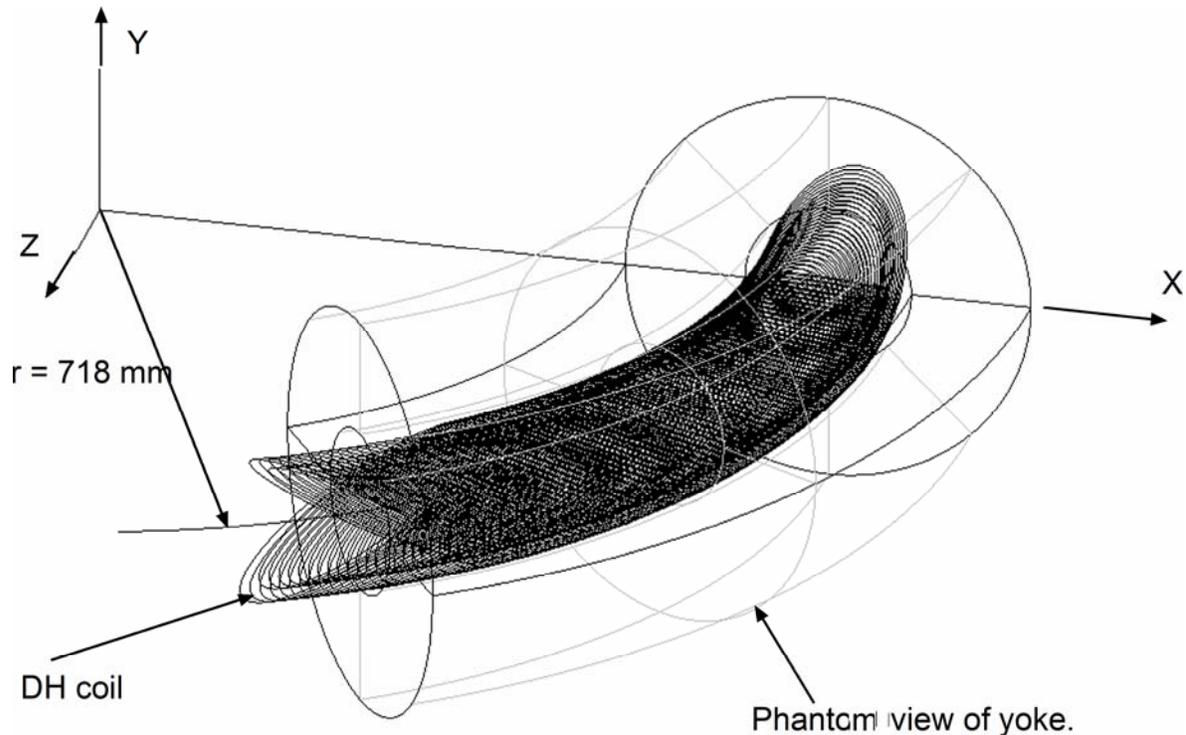
	Dipole	Dipole+quad
Coil aperture, mm	100	100
Coil current, A	5000	5000
Conductor turn spacing, mm	6.400	11.421
Quadrupole amplitude (e2)	0.0	0.5
Dipole field, T	-5.124	-3.047
Quadrupole field @ r=30 mm, T	0.000	-1.385
Gradient, T/m	0.000	-46.167

It is seen that in order to obtain the 50% quadrupole content, the effective conductor spacing has been increased from 6.4 to 11.421 mm, thus decreasing the transfer function of the magnet.

### 2.3. Correcting Field Errors with Combined Function Double-Helix Coils

#### 2.3.1. Quadrupole correction in bent coils.

Dipoles with small bend radii are easy to manufacture using double-helix coils, and the inherent quadrupole that is produced by bending a dipole is easily compensated by using the combined function feature of double-helix coils. The example shown here is a proposed dipole with a bend radius of 718 mm and an aperture of 100 mm. This type of magnet is being considered for future rare isotope accelerators (i.e. AEBL and RIA) that require special bending and focusing magnets. The magnetic system for beam transport requires a 180° bend of the beam channel, which is accomplished with a set of 4 bent dipole magnets. The coil configuration shown in Figure 5 represents the winding pattern of the coils for a 4.5 T dipole with a bend radius of 718 mm and an arc of 55°.



**Figure 5. Diagram of a proposed 4-layer double-helix coil used in a 180° beam channel bend.**

An analysis of the field produced by this configuration (without iron) shows a significant quadrupole ( $\sim 260$  units of  $10^{-4}$  of the main field). From the relationship shown in Equation 2 it can be shown that the units of quadrupole produced in the bent coil vary linearly with the quadrupole modulation function,  $\epsilon_2$ , as shown in Figure 6. It is seen that when  $\epsilon_2 = \sim 0.059$ , the component vanishes. Applying this value of modulation in the coil winding produces the result shown in Table 2. This demonstrates that the combined function capability of this coil design can tune out the quadrupole independently of the other harmonics and thus solve this difficult problem. This procedure can then be applied to the complete magnet with yoke to tune out the quadrupole and other saturation induced multipoles as required.

**Table 2. Compensation of quadrupole component in a bent magnet (without yoke) by adjusting the modulation amplitude,  $\epsilon_2$ , in Equation 1. Quadrupole component is shown as units of  $10^{-4}$  of the dipole field at a reference radius of 30 mm for the 100 mm aperture coil.**

Conductor path modulation from (Equation 1)	Bent coils (R=718 mm)	
	$B_0$ , T	Quadrupole (units)
$z(\theta) = A_0 (\sin \theta)$	-3.17	258.64
$z(\theta) = A_0 (\sin \theta + 0.05896 \sin(2\theta))$	-3.17	1.36

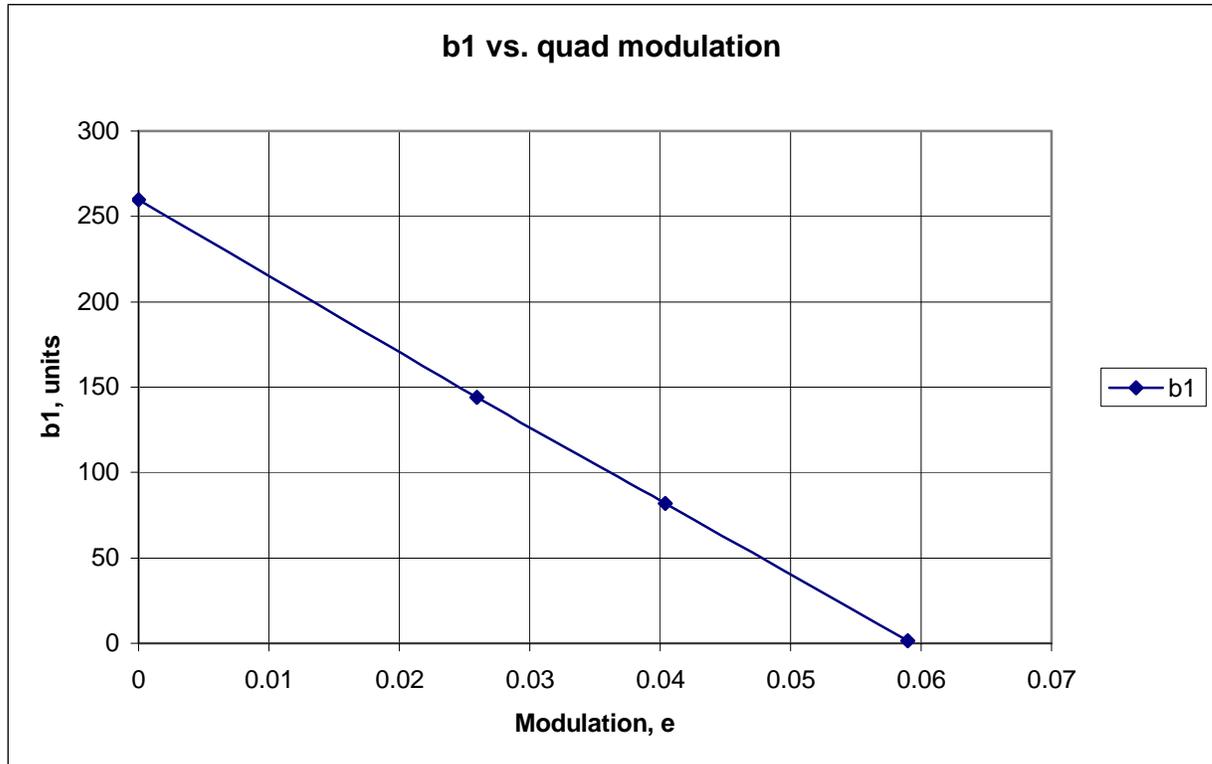


Figure 6. Units of quadrupole produced in the bent coils of Figure 5 (without iron) as a function of the quadrupole modulation factor ( $\epsilon_2$ ) of the coil.

### 2.3.2. Correcting saturation induced multipoles.

Although the double-helix coil inherently produces a pure multipole field, harmonics of dipole symmetry are produced if an iron yoke surrounds the coils, especially in magnets at higher field levels. An example is given here of how the use of **Equation 1** can be used to modify the coil winding pattern of double helix coils to eliminate saturation induced multipoles in a dipole magnet.

In this example we take the case of the four layer dipole coil used in Table 1. A linear periodic model used in this study is shown in Figure 7 and the results of this study are summarized in Table 3.

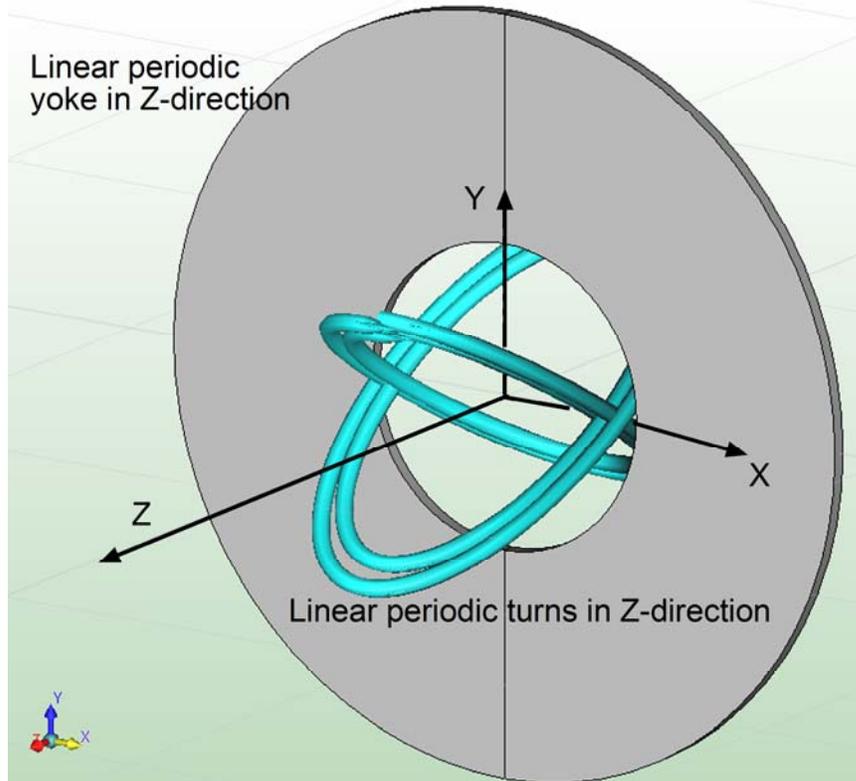


Figure 7. Linear periodic model of a 4-layer double-helix coil in an iron yoke.

Table 3. Analysis of compensated double-helix coils with modulation amplitudes  $\epsilon_3 = 3.5600\text{E-}03$  and  $\epsilon_5 = 1.5338\text{E-}03$  used in Equation 1. Harmonics are shown at a reference radius of 30 mm in units of  $10^{-4}$  of the dipole field.

Conductor path modulation from (Equation 1)		Basic coil			Coil with yoke		
		$B_0$ , T	Sextupole (units)	Decapole (units)	$B_0$ , T	Sextupole (units)	Decapole (units)
1	$z(\theta) = A_0(\sin \theta)$	3.45	-0.03	0.00	5.12	23.08	-4.89
2	$z(\theta) = A_0(\sin \theta + \epsilon_3 \sin(3\theta))$	3.40	-31.22	0.00	5.12	0.48	-4.93
3	$z(\theta) = A_0(\sin \theta + \epsilon_5 \sin(5\theta))$	3.40	-0.01	6.72	5.12	23.25	-0.25
4	$z(\theta) = A_0(\sin \theta + \epsilon_3 \sin(3\theta) + \epsilon_5 \sin(5\theta))$	3.40	-31.22	6.72	5.12	0.17	-0.36

This result shows that with the plain yoke shown in Figure 7, the dipole field is enhanced by 50% and the saturation-induced sextupole and decapole are 23 and -4.9 units (Row 1). In Row 2, the coil has been modulated with a 3.56E-03 sextuple amplitude. This produces -31 units (relative to 3.4 T) for the coil without the yoke. This compensates the 23 units of saturation sextupole (relative to 5.12 T) as shown. However, the -4.9 units of decapole remain. Row 3 shows what happens when a factor of 1.5338E-03 is applied to the decapole amplitude. The decapole is virtually canceled and the sextupole remains unchanged. In Row 4, both corrections

for  $\varepsilon_3$  and  $\varepsilon_5$  have been applied to the coil path, and the result is that both the sextupole and decapole have been virtually tuned out.

### 3. SUMMARY

The double-helix coil geometry presents a powerful method of modulating the conductor path to produce magnets having combined function characteristics that are easy to manufacture at a low cost. Three examples have been shown:

1. A combined function dipole with a 50 % quadrupole component. The limitation of the amount of dipole transfer function that can be obtained with a specified level of quadrupole has been explained.

2. Elimination of the inherent quadrupole component that is produced in a bent magnet of relatively small radius.

3. Elimination of saturation-induced multipoles due to the presence of an iron yoke in dipole magnets. This case also demonstrated how each harmonic can be tuned independently of the other harmonics present in the field.

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[1] R.B. Meinke, M.J. Ball, C.L. Goodzeit, "Superconducting Double-Helix Accelerator Magnets", IEEE Proceedings of the 2003 Particle Accelerator Conference, 2003, Vol.3, pages 1996-1998.

[2] C. L. Goodzeit, M. J. Ball, R. B. Meinke, "The Double-Helix Dipole – A Novel Approach to Accelerator Magnet Design", IEEE Transactions on Applied Superconductivity, June 2003, Vol.13, No. 2, pp. 1365-1368.

[3] R. B. Meinke, C. L. Goodzeit, M. J. Ball, "Modulated Double-Helix Quadrupole Magnets", IEEE Transactions on Applied Superconductivity, June 2003, Vol.13, No. 2, pp. 1369-1372.